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# A kind of prediction from superstring model building

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**ABSTRACT:** Assuming that the Standard Model of particle physics arises from the  $E_8 \times E_8$  Heterotic String Theory, we try to solve the discrepancy between the unification scale predicted by this theory ( $\approx g_{\text{GUT}} \times 5.27 \cdot 10^{17} \text{ GeV}$ ) and the value deduced from LEP experiments ( $\approx 2 \cdot 10^{16} \text{ GeV}$ ). A crucial ingredient in our solution is the presence at low energies of three generations of supersymmetric Higgses and vector-like colour triplets. As a by-product our analysis gives rise to a strategy which might be useful when constructing realistic models.

**KEYWORDS:** Superstrings and Heterotic Strings, Superstring Vacua, Compactification and String Models.

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## 1. Introduction

The Standard Model of particle physics [1] provides a correct description of the observable world. However, there exist strong indications that it is just a low-energy effective theory. There is no answer in the context of the Standard Model to some fundamental questions. For example, How can we unify it with gravity? And then: How can we protect the masses of the scalar particles against quadratic divergences in perturbation theory (the so-called hierarchy problem)? Other questions cannot even be posed: Why is the Standard Model gauge group  $SU(3) \times SU(2) \times U(1)_Y$ ? Why are there three families of particles? Why is the pattern of quark and lepton masses so weird?

Supersymmetry [2] is an interesting step in answering some of these questions. In addition to introducing a kind of unification between particles of different spin, it also contributes to solving the hierarchy problem of the Standard Model. It ensures the stability of the hierarchy between the weak and the Planck scale. Furthermore, its local version, Supergravity [3], leads to a partial unification of the Standard Model with gravity. However, only String Theory has the potential to unify all gauge interactions with gravity [4] in a consistent way [5]. In fact Supergravity is the low-energy limit of (Super)string Theory. In this sense Superstring Theory would be the fundamental theory of particle physics which, in principle, might be able to answer all the above questions. Its detailed analysis is therefore very important.

One crucial step in this analysis consists in achieving contact between the theory and the low-energy world. We need to find a consistent Superstring Theory in four dimensions which is able to accommodate the observed Standard Model of particle physics, i.e. we need to find the Superstring Standard Model. In the late eighties, the compactification of the  $E_8 \times E_8$  Heterotic String [6] on six-dimensional orbifolds [7] proved to be an interesting method to carry out these tasks [8]–[24] (other attempts at model building, using Calabi-Yau spaces [25] and fermionic constructions [26], can be found in refs. [27] and [28]–[30], respectively).

It was first shown that the use of background fields (Wilson lines) [7, 8] on the torus defining the symmetric  $Z_3$  orbifold can give rise to four-dimensional supersymmetric models with gauge group  $SU(3) \times SU(2) \times U(1)^5 \times G_{\text{hidden}}$  and three generations of chiral particles with the correct  $SU(3) \times SU(2)$  representations (plus many extra particles) [9, 11]. In fact, it was also shown that the three generations appear in a natural way using just two discrete Wilson lines. This is so because in addition to the overall factor of 3 coming from the right-moving part of the untwisted matter, the twisted matter come in 9 sets with 3 equivalent sectors on each one, since there are 27 fixed points. In this way, all matter (including extra particles) in these constructions appear automatically with three generations.

The next step was the calculation of the  $U(1)$  charges and the study of the mechanism for anomaly cancellation in these models [12], since an anomalous  $U(1)$  is usually present after compactification [31]. This allowed the construction of combinations of the non-anomalous  $U(1)$ 's giving the physical hypercharge for the particles of the Standard Model, although it was also found that the hidden sector is, in general, mixed with the observable one through the extra  $U(1)$  charges. Fortunately, it was also noted that the Fayet-Iliopoulos D-term [31], which appears because of the presence of the anomalous  $U(1)$ , can give rise to the breaking of the extra  $U(1)$ 's and, as a consequence, to the hiding of the previously mixed hidden sector [12, 13, 15]. This is because, in order to preserve supersymmetry at high energies, some scalars with  $U(1)$ 's quantum numbers acquire large vacuum expectation values (VEVs). It is worth noticing here that the Fayet-Iliopoulos D-term was also proposed in order to produce inflation in these models [32].

In this way it was possible to construct supersymmetric models (or, more precisely, vacuum states) where the original  $SU(3) \times SU(2) \times U(1)^5 \times SO(10) \times U(1)^3$  gauge group [9] was broken to  $SU(3) \times SU(2) \times U(1)_Y \times SO(10)_{\text{hidden}}$  with three generations of particles in the observable sector with the correct quantum numbers [16, 17, 12]. In addition, baryon- and lepton-number-violating operators are absent. Unfortunately, we cannot claim that one of these models is the Superstring Standard Model. It is true that, as we will discuss below, the initially large number of extra particles is highly reduced since many of them get a high mass ( $\approx 10^{16-17}$  GeV) through the Fayet-Iliopoulos mechanism, thus disappearing from the low-energy theory [16, 17, 12]. However, in general, some extra  $SU(3)$  triplets,  $SU(2)$  doublets and  $SU(3) \times SU(2)$  singlets still remain. On the other hand, given the predicted value for the unification scale in the Heterotic String [33],  $M_{\text{GUT}} \approx g_{\text{GUT}} \times 5.27 \cdot 10^{17}$  GeV, with  $g_{\text{GUT}}$  the unified gauge coupling, the values of the gauge couplings deduced from CERN  $e^+e^-$  collider LEP experiments cannot be obtained [19]. Finally, it was not possible to obtain correct Yukawa couplings in these models [12, 17, 18].

In any case, it is plausible to think that another orbifold model could be found with the right properties. In the present paper we will adopt this point of view. We will assume that the Supersymmetric Standard Model arises from the Heterotic String compactified on a  $Z_3$  symmetric orbifold with two Wilson lines. Then, we will try to deduce the phenomenological properties that such a model must have in order to solve the first and second problems mentioned above, namely extra matter and gauge coupling unification.<sup>1</sup> In fact, the two problems are closely related, since the evolution of the gauge couplings from high

<sup>1</sup>As a matter of fact, our arguments will be general and can be applied to any scheme giving rise to

to low energy through the renormalization group equations depends on the existing matter [34, 35]. With our solution we will be able to predict the existence of three generations of supersymmetric Higgses and vector-like colour triplets at low energies. As a by-product, a strategy to construct the sought-after Superstring Standard Model will arise.

Concerning the third problem, how to obtain the observed structure of fermion masses and quark mixing angles, this is the most difficult task in string model building, and beyond the scope of this paper. However, we will mention it briefly in the conclusions. Needless to say, the experimental confirmation of neutrino masses in the near future will make this task even more involved.

Finally, before starting with our computation, let us mention that in the late nineties it has been discovered that explicit models, with interesting phenomenological properties, can also be constructed using D-brane configurations from type I string vacua [36]. It might well be possible that the Superstring Standard Model arose from one of them. However, in our opinion, those models are still not as satisfactory as the (already fourteen years old) heterotic ones described above.

## 2. Scales of the theory

Since we are interested in the analysis of gauge couplings, we need to first clarify which are the relevant scales for the running between the mass of the  $Z$ ,  $M_Z$ , and the unification point. Two of them are quite clear in heterotic compactifications. We have first the different thresholds associated to the masses of the supersymmetric particles. For the sake of simplicity we will assume that supersymmetry remains unbroken for energies above 500 GeV, and we will use this value as our overall supersymmetric scale  $M_S$ . In fact, we will use a similar approximation for the thresholds due to the top quark and light Higgs doublet, since we will take  $M_Z$  as our overall non-supersymmetric scale. The second relevant scale is associated to the Fayet-Iliopoulos mechanism to be discussed now.

As mentioned in the Introduction, some scalars, in particular  $SU(3) \times SU(2)$  singlets, develop VEVs in order to cancel the Fayet-Iliopoulos D-term. This is given by [31]:

$$D^{(a)} = \sum_i Q_i^{(a)} \eta_i \frac{\partial K}{\partial \eta_i} + \frac{g_{\text{GUT}}^2 \text{tr} Q^{(a)}}{192\pi^2} M_P^2, \quad (2.1)$$

where  $\eta_i$  are the scalar fields with charges  $Q_i^{(a)}$  under the anomalous  $U(1)$ ,  $K$  is the Kähler potential (e.g. considering an overall modulus  $T$ ,  $K = (T + \bar{T})^{-n} \eta_i \eta_i^*$  with  $n = 1, 2, 3$  for untwisted, twisted non-oscillator and twisted oscillator fields, respectively), and  $M_P = M_{\text{Planck}}/\sqrt{8\pi}$  is the reduced Planck mass. Obviously  $\text{tr} Q^{(a)} = 0$  if the model does not have any anomalous  $U(1)$ , a situation that is not very common in  $Z_3$  orbifold constructions, as we will see below. Since we want to have a vacuum state preserving the physical hypercharge  $Y$ , we have to look for the subset of singlet fields with vanishing  $Y$ -charges. Remarkably enough, in all constructed models there is a large subset of singlets, say  $\chi_j$ , with  $Y = 0$ .

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three generations, since extra matter and anomalous  $U(1)$ 's are generically present in compactifications of the Heterotic String.

Although their VEVs are model dependent, as we can see from eq. (2.1), an estimate can be done with the average result  $\langle \chi_j \rangle \sim 10^{16-17}$  GeV (see e.g. ref. [35]). After the breaking, many particles, say  $\xi$ , acquire a high mass because of the generation of effective mass terms. These come for example from operators of the type  $\chi_j \xi \xi$ . In this way vector-like triplets and doublets and also singlets become very heavy. We will see that this is the type of extra matter that typically appears in orbifold constructions. Again, for the sake of simplicity, we will use the above value as our overall Fayet-Iliopoulos scale  $M_{FI} \approx 10^{16-17}$  GeV.

In principle, other thresholds might appear in these Heterotic String constructions. These would be due to the possible presence of higher order operators. For example, we might have terms in the superpotential of the type  $\frac{1}{M_P^{m-1}} \chi_1 \cdots \chi_m \xi \xi$ , which would produce masses of the order of  $(M_{FI}/M_P)^{m-1} M_{FI}$ . Therefore, depending on  $m$ , intermediate scale masses might be generated. Obviously, the presence of particles with these masses is very model-dependent and introduces a high degree of uncertainty in the computation. However, it is important to remark that the presence of the above non-renormalizable couplings is not always allowed in string constructions. First of all, they must be gauge-invariant, something that is not easy, because of the large number of U(1) charges associated to the particles in these models. Even if the couplings fulfil this condition, this does not mean that they are automatically allowed. They must still fulfil the so-called ‘stringy’ selection rules [37]. For example in the  $SU(3) \times SU(2) \times U(1)_Y \times SO(10)_{\text{hidden}}$  model of ref. [16], whereas a large number of renormalizable couplings are present, generating Fayet-Iliopoulos scale masses  $M_{FI}$  for the extra matter, only a small number of non-renormalizable couplings are allowed by gauge invariance. Moreover, at the end of the day, the latter are forbidden by the selection rules.

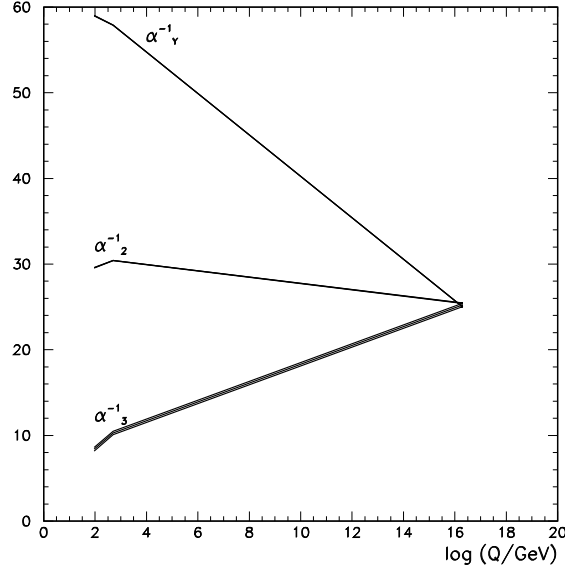
Taking into account the above comments, we will adopt in this paper the following point of view: all the extra matter in the  $Z_3$  orbifold models to be analysed is massless or very heavy (with masses of the order of  $M_{FI}$ ). In the former situation they must acquire masses through the electroweak symmetry breaking, as we will discuss below. In any case, since e.g. no new quarks have been observed in colliders, their masses must be basically heavier than 200 GeV. Again, in order to simplify the analysis, we will consider that the masses of these extra particles are of the order of  $M_S$ .

### 3. Analysis of the RGEs for gauge couplings

Let us turn now to the details of the calculation. The one-loop runnings of the gauge couplings with energy  $Q$  are

$$\frac{1}{\alpha_i(Q)} = \frac{1}{\alpha_i(M_Z)} + \frac{b_i^{NS}}{2\pi} \ln \frac{M_S}{M_Z} + \frac{b_i^S}{2\pi} \ln \frac{M_{FI}}{M_S} + \frac{b_i^{FI}}{2\pi} \ln \frac{Q}{M_{FI}}, \quad (3.1)$$

where  $\alpha_i = g_i^2/4\pi$  with  $i = 2, 3, Y$ , and  $b_i$ ’s are the coefficients of the  $\beta$ -functions. In particular, using the matter content of the standard model (with one Higgs doublet), the non-supersymmetric  $\beta$ -functions are given by  $b_3^{NS} = 7$ ,  $b_2^{NS} = 19/6$  and  $b_Y^{NS} = -C^2 \times 41/6$ . As we will discuss in detail below, the normalization constant,  $C$ , of the  $U(1)_Y$  hypercharge generator is not fixed as in the case of grand unified theories (e.g. for  $SU(5)$ ,  $C^2 = 3/5$ ),



**Figure 1:** Unification of the gauge couplings of the MSSM at  $M_{\text{GUT}} \approx 2 \cdot 10^{16}$  GeV, using  $C^2 = 3/5$  as the normalization factor for the hypercharge.

so for the moment we consider it as a free parameter. The supersymmetric  $\beta$ -functions between the supersymmetric scale  $M_S$  and the Fayet-Iliopoulos scale  $M_{FI}$ , considering three supersymmetric generations of standard particles, two Higgs doublets and an arbitrary number of extra particles are

$$b_3^S = 3 - \frac{1}{2}n_3, \quad (3.2)$$

$$b_2^S = -1 - \frac{1}{2}n_2, \quad (3.3)$$

$$b_Y^S = -C^2 \times (11 + q), \quad (3.4)$$

where

$$q = \sum_{i=1}^{n_1} Y_i^2 + 2 \sum_{j=1}^{n_2} Y_j^2 + 3 \sum_{k=1}^{n_3} Y_k^2, \quad (3.5)$$

and  $n_1, n_2, n_3$  is the number of extra  $\text{SU}(3) \times \text{SU}(2)$  singlets,  $\text{SU}(2)$  doublets and  $\text{SU}(3)$  triplets, respectively, with masses close to  $M_S$  and hypercharges  $Y_l$ . Finally, the supersymmetric  $\beta$ -functions beyond  $M_{FI}$ , considering an arbitrary number of extra singlets,  $n_1^{FI}$ , doublets,  $n_2^{FI}$ , and triplets,  $n_3^{FI}$ , all of them with masses close to  $M_{FI}$ , are

$$\begin{aligned} b_3^{FI} &= b_3^S - \frac{1}{2}n_3^{FI}, \\ b_2^{FI} &= b_2^S - \frac{1}{2}n_2^{FI}, \\ b_Y^{FI} &= b_Y^S - C^2 \times (q^{FI}), \end{aligned} \quad (3.6)$$

where  $q^{FI}$  is given by eq. (3.5) with the substitution  $n_{1,2,3} \rightarrow n_{1,2,3}^{FI}$ .

It is worth remarking here that moduli-dependent string threshold corrections to eq. (3.1) are absent in the case of the  $Z_3$  orbifold [33]. Moduli-independent ones are generically present but, although model-dependent, they are small [33, 38] and we neglect them in the computation.

Now, using for example the above equations with  $n_3 = n_2 = q = n_3^{FI} = n_2^{FI} = q^{FI} = 0$ , one is able to reproduce straightforwardly the well-known prediction of the Minimal Supersymmetric Standard Model (MSSM), namely the three gauge couplings unify (*assuming* the normalization constant of  $SU(5)$ ) at  $M_{\text{GUT}} \approx 2 \cdot 10^{16} \text{ GeV}$ . This is shown in figure 1, using the experimental values [39]  $M_Z = 91.187 \text{ GeV}$ ,  $\alpha_3^{-1}(M_Z) = 8.39 \pm 0.2$ ,  $\alpha_2^{-1}(M_Z) = 29.567 \pm 0.02$  and  $\alpha_Y^{-1}(M_Z) = C^2 \times (98.333 \pm 0.02)$ , which are given in the  $\overline{MS}$  scheme. We neglect the conversion factors to the usual  $\overline{DR}$  supersymmetric scheme since they are very small (see e.g. [40]). In addition we are neglecting higher-loop corrections to the running (for an estimate of these small corrections see [41]). The results shown in this and the other figures in this paper are not going to be modified qualitatively by these (small) effects. We have checked it explicitly (adding also moduli-independent string threshold corrections). For instance, even if two-loop effects spoiled the unification, they could be counteracted by adjusting e.g. the scale  $M_S$ .

As discussed in the Introduction, we are interested in the unification of the gauge couplings at  $M_{\text{GUT}} \approx g_{\text{GUT}} \times 5.27 \cdot 10^{17} \text{ GeV}$ . This is not a simple issue, and various approaches towards understanding it have been proposed in the literature [42]. Some of these proposals consist of using string GUT models, extra matter at intermediate scales, heavy string threshold corrections, non-standard hypercharge normalizations, etc. In our case, we will try to obtain this value by using first the existence of extra matter at the scale  $M_S$ . We will see that this is not sufficient and more ideas must be involved.

### 3.1 Predictions from the unification of $\alpha_3$ with $\alpha_2$

Let us concentrate for the moment on  $\alpha_3$  and  $\alpha_2$ , neglecting the scale  $M_{FI}$ . Recalling that three generations appear automatically for all the matter in  $Z_3$  orbifold scenarios with two Wilson lines, the most natural possibility is to assume the presence of three light generations of supersymmetric Higgses. This implies that we have four extra Higgs doublets with respect to the case of the MSSM and therefore we have to use  $n_2 = 4$  and  $n_3 = 0$  in eqs. (3.3) and (3.2), respectively. Unfortunately, this goes wrong. Whereas  $\alpha_3^{-1}$  remains unchanged, the line of figure 1 for  $\alpha_2^{-1}$  is pushed down (see eq. (3.1)). As a consequence, the two couplings cross at a very low scale ( $\approx 10^{12} \text{ GeV}$ ). We could try to improve this situation by assuming the presence of extra triplets in addition to the four extra doublets. Then the line for  $\alpha_3^{-1}$  is also pushed down and therefore the crossing might be obtained for larger scales. However, even for the minimum number of extra triplets that can be naturally obtained in our scenario,  $3 \times \{(3, 1) + (\bar{3}, 1)\}$ , i.e.  $n_3 = 6$ , the “unification” scale turns out to be too large ( $\approx 10^{21} \text{ GeV}$ ). The next simplest possibility,  $n_2 = 7$  and  $n_3 = 6$ , produces the crossing at the scale  $\approx 10^{15} \text{ GeV}$ , again well below the required value, as in our first attempt. More extra triplets would imply at least  $n_3 = 12$  and therefore  $\alpha_3^{-1}$  becomes negative at the scale  $\approx 10^{13} \text{ GeV}$ . Summarizing, using extra matter at  $M_S$  we are not able to obtain the Heterotic String unification scale since  $\alpha_3$  never crosses  $\alpha_2$

at  $M_{\text{GUT}} \approx g_{\text{GUT}} \times 5.27 \cdot 10^{17} \text{ GeV}$ . Fortunately, this is not the end of the story. As we will show now, the Fayet-Iliopoulos scale  $M_{FI}$  is going to play an important role in the analysis.

Using eq. (3.1) with  $i = 3, 2$ , and imposing  $\alpha_3(Q = M_{\text{GUT}}) = \alpha_2(Q = M_{\text{GUT}})$ , one obtains the following value for the unification scale, taking into account  $M_{FI}$ :

$$\ln \frac{M_{\text{GUT}}}{M_{FI}} = \frac{4\pi [\alpha_2^{-1}(M_Z) - \alpha_3^{-1}(M_Z)] - (23/3) \ln(M_S/M_Z) - [8 + (n_2 - n_3)] \ln(M_{FI}/M_S)}{[8 + (n_2 - n_3) + (n_2^{FI} - n_3^{FI})]}. \quad (3.7)$$

In order to determine whether or not the Heterotic String unification scale can be obtained, we need to know the number of doublets and triplets in our construction with masses of the order of the Fayet-Iliopoulos scale  $M_{FI}$  (in fact knowing the difference between the number of doublets and the number of triplets,  $n_2^{FI} - n_3^{FI}$ , is enough). Let us explain this calculation in some detail.

In principle one can construct, within the  $Z_3$  orbifold with two Wilson lines, a number of the order of 50000 of three-generation models with the  $\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)^5$  gauge group associated to the first  $E_8$  and the  $(3, 2)$  matter representation in the untwisted sector (in case of the  $(3, 2)$  representation in the twisted sector,  $\text{SU}(3) \times \text{SU}(3) \times \text{U}(1)^4$  is the smallest gauge group that can be obtained associated to the first  $E_8$  [23]). However, a detailed analysis implied that most of them are equivalent. In fact, at the end of the day, only 9 models are left [21]. The (observable) untwisted matter associated with the first  $E_8$  is uniquely determined and can be easily computed. There are only two types. Considering just the  $\text{SU}(3)$  triplets and  $\text{SU}(2)$  doublets, these are  $3 \times \{(3, 2)\}$  and  $3 \times \{(3, 2) + (\bar{3}, 1) + (1, 2)\}$  [21]. Thus there are three more doublets than triplets in both cases. The determination of the twisted matter of these models on a model-by-model basis is more involved, since one has to add several hidden-sector parts associated to the second  $E_8$ . This is necessary even for the twisted matter associated to the observable sector. However, taking into account the result for the untwisted matter, duality-anomaly cancellation arguments show that there must be nine more doublets than triplets in the observable twisted sector [43]. Altogether one obtains that there are twelve more doublets than triplets in all models. Since the Standard Model (excluding the Higgs sector) contains the same number of doublets as triplets,  $3 \times \{(3, 2) + 2(\bar{3}, 1) + (1, 2)\}$ , we obtain the relation  $2 + n_2 + n_2^{FI} = n_3 + n_3^{FI} + 12$ . It is now straightforward to check, using eq. (3.7), that only models with

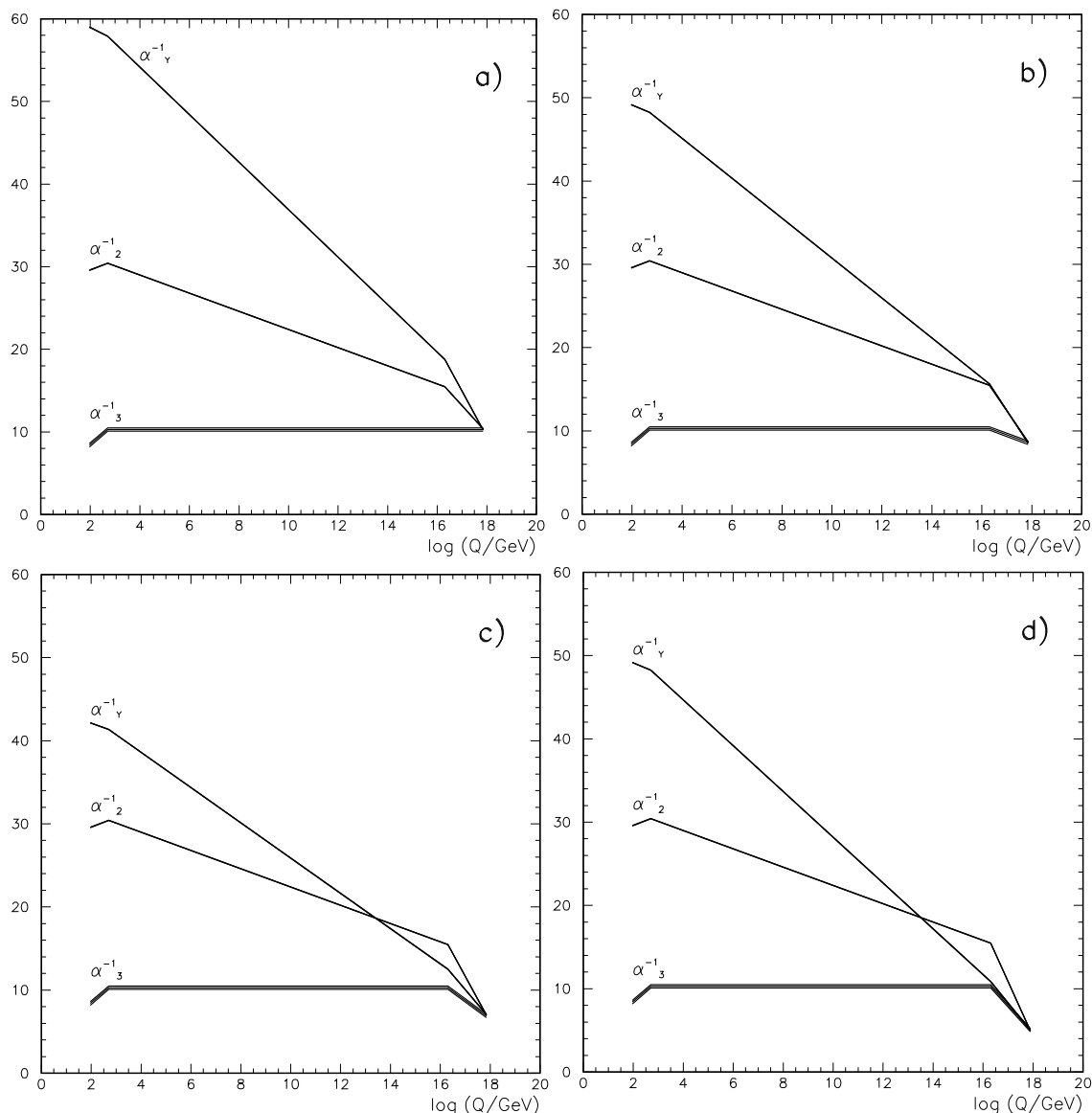
$$n_2 = 4, \quad n_3 = 6, \quad (3.8)$$

and therefore  $n_2^{FI} - n_3^{FI} = 12$ , may give rise to the Heterotic String unification scale (the other possibilities for  $n_2, n_3$ , discussed above do not even produce the crossing of  $\alpha_3$  and  $\alpha_2$ ). This is shown in figure 2. There we are using  $M_{FI} = 2 \cdot 10^{16} \text{ GeV}$  as will be discussed below. We also postpone for the moment the discussion about the coupling  $\alpha_1$ .

Note that at low energy we then have (excluding singlets)

$$3 \times \{(3, 2) + 2(\bar{3}, 1) + (1, 2)\} + 3 \times \{(3, 1) + (\bar{3}, 1) + 2(1, 2)\}, \quad (3.9)$$





**Figure 2:** Unification of the gauge couplings at  $M_{\text{GUT}} \approx g_{\text{GUT}} \times 5.27 \cdot 10^{17} \text{ GeV}$  for the Heterotic String construction analysed in the text with three light generations of supersymmetric Higgses and vector-like colour triplets,  $n_2 = 4$ ,  $n_3 = 6$ . Cases a), b), c) and d) correspond to the four possible patterns of heavy matter in eq. (3.10). For each case, the line corresponding to  $\alpha_1$  is just one example of the many possibilities discussed below eq. (3.11).

i.e. the matter content of the Supersymmetric Standard Model with three generations of Higgses and vector-like colour triplets. In section 4 we will discuss in some detail the phenomenology associated to this scenario, where extra matter at low energy is present.

Let us remark that although in this scenario  $M_{\text{GUT}}$  only depends on the difference between the number of doublets and the number of triplets  $n_2^{FI} - n_3^{FI}$ , the value of the unified coupling constant  $g_{\text{GUT}}$  indeed depends on the precise number of matter multiplets. For example, using eq. (3.1) with  $i = 2$  we need to know  $n_2^{FI}$ . Since  $n_2^{FI} - n_3^{FI} = 12$ , the

following patterns of matter with masses of the order of  $M_{FI}$  are allowed:

$$\begin{aligned}
a) \quad n_3^{FI} &= 0, & n_2^{FI} &= 12 \rightarrow 3 \times \{4(1, 2)\}, \\
b) \quad n_3^{FI} &= 6, & n_2^{FI} &= 18 \rightarrow 3 \times \{(3, 1) + (\bar{3}, 1) + 6(1, 2)\}, \\
c) \quad n_3^{FI} &= 12, & n_2^{FI} &= 24 \rightarrow 3 \times \{2[(3, 1) + (\bar{3}, 1)] + 8(1, 2)\}, \\
d) \quad n_3^{FI} &= 18, & n_2^{FI} &= 30 \rightarrow 3 \times \{3[(3, 1) + (\bar{3}, 1)] + 10(1, 2)\}.
\end{aligned} \tag{3.10}$$

Patterns with 24 or more colour triplets (and the corresponding SU(2) doublets) can be discarded. The reason is the following. Recently, the analysis of ref. [21] discussed above was extended in ref. [44]. There, the number of inequivalent models associated to the first  $E_8$  was reduced further and only 6 were left. In addition, when the second  $E_8$  was added in the analysis, only 192 different models were found. They have only five possible hidden-sector gauge groups,  $SO(10) \times U(1)^3$ ,  $SU(5) \times SU(2) \times U(1)^3$ ,  $SU(4) \times SU(2)^2 \times U(1)^3$ ,  $SU(3) \times SU(2)^2 \times U(1)^4$ ,  $SU(2)^2 \times U(1)^6$ . Then, the matter content of the 175 models associated to the first four gauge groups was analysed in detail [35]. It is straightforward to obtain from that classification that for 162 of those models only the above four patterns are present. For the rest, either they have no anomalous U(1) associated (7 of them), and therefore the Fayet-Iliopoulos mechanism does not work, or they have no extra triplets at all,  $n_3 = n_3^{FI} = 0$  (6 of them).

For a given Fayet-Iliopoulos scale,  $M_{FI}$ , each one of the four patterns in eq. (3.10) will give rise to a different value for  $g_{GUT}$ . Adjusting  $M_{FI}$  appropriately, we can always get  $M_{GUT} \approx g_{GUT} \times 5.27 \cdot 10^{17} \text{ GeV}$ . In particular this is so for  $M_{FI} \approx 2 \times 10^{16} \text{ GeV}$  as shown in figure 2. It is remarkable that this number is within the allowed range for the Fayet-Iliopoulos breaking scale discussed below eq. (2.1). We also see in figure 2 that patterns a), b), c) and d) have  $g \approx 1.1$ ,  $g \approx 1.2$ ,  $g \approx 1.3$  and  $g \approx 1.5$ , respectively, and therefore  $M_{GUT} \approx 5.8 \cdot 10^{17} \text{ GeV}$ ,  $M_{GUT} \approx 6.3 \cdot 10^{17} \text{ GeV}$ ,  $M_{GUT} \approx 6.8 \cdot 10^{17} \text{ GeV}$  and  $M_{GUT} \approx 7.9 \cdot 10^{17} \text{ GeV}$ .

### 3.2 Analysis of $\alpha_1$

Of course, we cannot claim to have obtained the Heterotic String unification scale until we have shown that the coupling  $\alpha_1$  joins the other two couplings at  $M_{GUT}$ . The analysis becomes more involved now because we need to know not only the hypercharges of the extra doublets and triplets of our scenario but also the ones of the extra singlets. In particular the latter are present in large numbers in these models  $\sim 3 \times (25-60)$ . Thus the analysis has to be carried out on a model-by-model basis. In addition, as already mentioned above, the normalization constant  $C$  of the U(1) hypercharge generator is not fixed as in the case of grand unified theories. In string constructions, the correct hypercharge for the physical particles is obtained as a combination of U(1)'s [12],  $Y = \sum c_i U_i$ , and therefore the normalization factor is given by  $C = (\sum c_i^2)^{-1/2}$  [19]. This means that the combination for the hypercharge depends on the model and, even for a specific model, there exist many acceptable combinations [12].

The only model-independent information we have concerning the value of  $C$  is the upper bound found in ref. [19] for the case of  $Z_3$  orbifold compactifications, namely  $C \leq 1$ .

On the other hand, the normalization factors of the 175 models mentioned above were analysed in ref. [35] and it seems that only  $C \lesssim \sqrt{3/5}$  may be obtained.

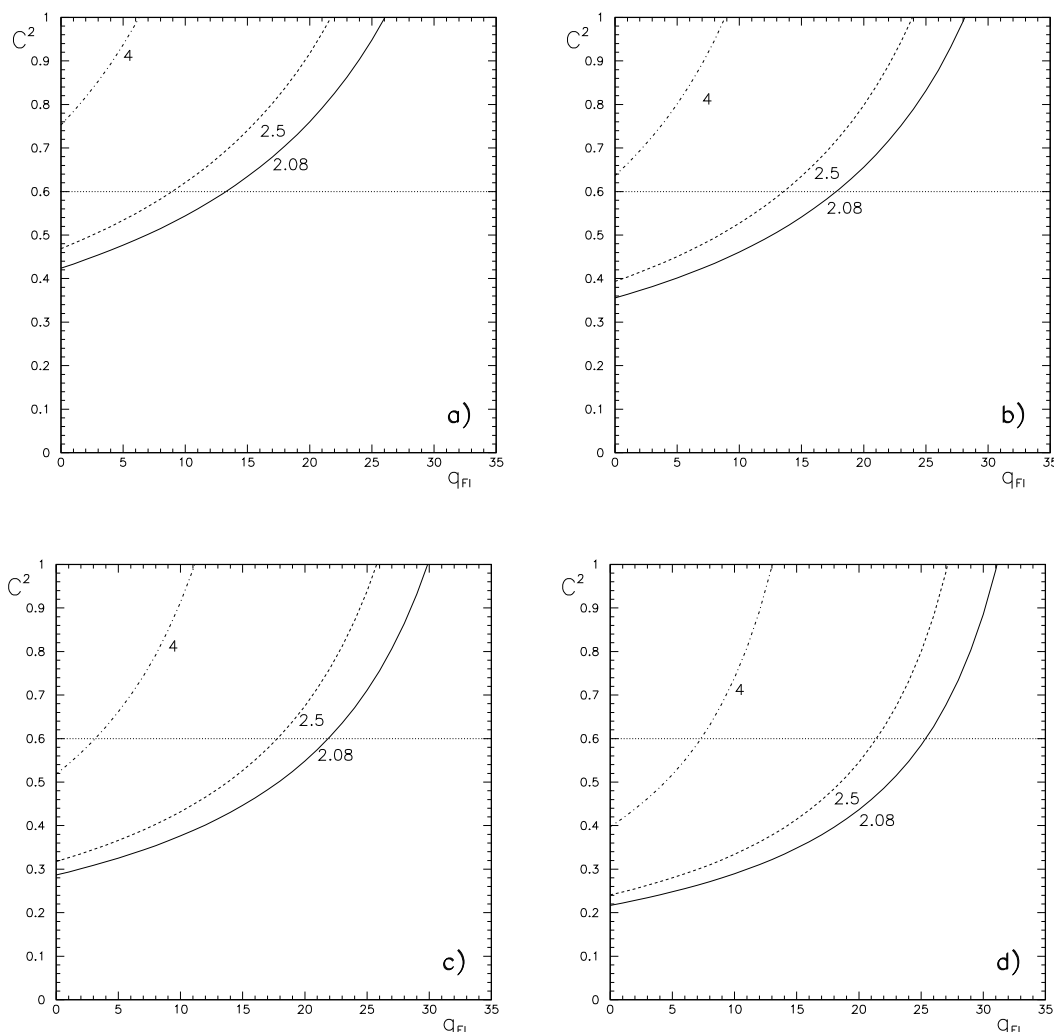
Then, what we will try to do is to study whether or not it is plausible, in these models, to obtain the correct value for  $\alpha_1$  at  $M_{\text{GUT}}$ . Let us analyse first the value of  $q$  in eq. (3.5). We know that the four extra doublets contribute with  $q = 2$ , since they are Higgses with hypercharges  $\pm 1/2$ . On the other hand, we cannot know, without a model-by-model analysis, the hypercharges associated to the six extra triplets. The same argument applies to the extra singlets, which could also be present at  $M_S$ . However, concerning the latter, we have already argued above that, at least some of them, and in some explicit example all of them [17], will have vanishing hypercharge. For the computation of  $q^{FI}$  the situation is more uncertain, since we do not even know the hypercharges associated to the extra doublets. So the only thing we can say about  $q$  and  $q^{FI}$  is that they must fulfil the lower bounds  $q > 2$ ,  $q^{FI} > 0$ . Of course these bounds are very conservative. For example, if we assume that all extra singlets left at the scale  $M_S$  have  $Y = 0$ , some of the other singlets with masses close to the scale  $M_{FI}$  will have non-vanishing values, and therefore will contribute to  $q^{FI}$ .

Using eq. (3.1) with  $i = Y$  we can compute, for given values of  $q$  and  $q^{FI}$ , the appropriate value of  $C^2$  in order to unify the coupling  $\alpha_1$  with the others at the Heterotic String unification scale  $M_{\text{GUT}}$ :

$$C^2 = \frac{2\pi\alpha^{-1}(M_{\text{GUT}})}{2\pi \times 98.333 - (41/6)\ln(M_S/M_Z) - (11+q)\ln(M_{FI}/M_S) - (11+q+q^{FI})\ln(M_{\text{GUT}}/M_{FI})}. \quad (3.11)$$

The results are shown in figure 3, where three possible values for  $q$  are considered,  $q = 2.08, 2.5, 4$ , corresponding to six extra colour triplets (in addition to the four extra Higgs doublets) at  $M_S$ , say  $D$  and  $\overline{D}$ , with hypercharges  $Y = \pm 1/15, \pm 1/6, \mp 1/3$ , respectively. These hypercharges for triplets appear in the three  $Z_3$  orbifold models with two Wilson lines studied in detail in the literature. In particular, extra triplets with  $Y = \mp 1/3$  appear in the model of ref. [16], with  $Y = \pm 1/6$  (and also with  $Y = \mp 1/3$ ) in the model of ref. [17], and with  $Y = \pm 1/15$  in [35]. Triplets with hypercharge  $\pm 2/3$  also appear in the models of refs. [16, 17]. However, their contribution would imply a large value for  $q$ , in particular  $q = 10$ , and therefore no solution for positive  $C^2$  can be found, even with  $q^{FI} = 0$ . Thus it would be better to give, through the Fayet-Iliopoulos mechanism, heavy masses for those triplets (contributing only to  $q^{FI}$ ).

From figure 3a we obtain that pattern a) implies the following lower bound for the normalization constant:  $C^2 \gtrsim 3/7$ . For instance, for  $q = 2.08$  and  $q^{FI} = 12$  we need to have  $C^2 \approx 3/5$  in order to unify  $\alpha_1$  with the other couplings. This example is the one shown in figure 2a. The lower bound associated to pattern b) is  $C^2 \gtrsim 3/9$ . For the above example we would now need  $C^2 \approx 3/6$ , and this is shown in figure 2b. To pattern c) is associated the bound  $C^2 \gtrsim 3/11$ . For instance, for  $q = 2.5$  and  $q^{FI} = 9$  we need  $C^2 \approx 3/7$ . This is shown in figure 2c. Finally, for pattern d) the bound is  $C^2 \gtrsim 3/15$ . In figure 2d we show the case  $q = 4$ ,  $q^{FI} = 5$ , where  $C^2 \approx 3/6$  must be used.



**Figure 3:** Normalization factor of the hypercharge  $C^2$  versus  $q^{FI}$  in order to obtain the Heterotic String unification scale, for three different values of  $q$ . As discussed in the text we are using  $q = 2.08, 2.5, 4$ . The straight line indicates the usual SU(5) normalization,  $C^2 = 3/5$ . The results for the four possible patterns of heavy extra matter in eq. (3.10), a), b), c) and d), are shown.

The simplicity of the constraints that we have found is extremely useful in order to perform a systematic analysis of the phenomenological viability of all possible vacua. It is very likely that most of them can easily be discarded. Good examples are the above mentioned models [16, 17, 35]. In ref. [12] two possible U(1) combinations for the hypercharge were obtained for the model introduced in ref. [9]. One of them, with  $C^2 = 3/17$  [19], was studied in detail [16]. Assuming condition (3.8) after the Fayet-Iliopoulos breaking, the model corresponds to pattern d) in eq. (3.10), with  $q = 4$ ,  $q^{FI} = 99$ . We can check in figure 3d that there is no solution because of the large value of  $q^{FI}$ . In other words,  $C^2$  becomes negative using eq. (3.11), and therefore unification of the coupling  $\alpha_1$  with

the others is not possible. In ref. [17] the other combination with  $C^2 = 3/11$  was studied. Its analysis yields  $q = 2.5$ ,  $q^{FI} = 64.5$ . Again, because of the large value of  $q^{FI}$ , unification is not possible. Finally, let us consider the model studied in ref. [35] with  $C^2 = 15/37 \approx 0.4$ , which, assuming condition (3.8), would correspond to pattern a). Its analysis yields  $q = 2.08$ ,  $q^{FI} = 21.2$ . Now, as can be seen from figure 3a, a solution for  $C^2$  producing unification is possible, namely  $C^2 \approx 0.76$ . Unfortunately, the latter does not coincide with the normalization of the model written above. Furthermore, as discussed above,  $C^2$  is unlikely to be much bigger than 0.6.

It is important to remark again that the combination for the hypercharge is not unique in these orbifold constructions [12]. There are a lot of choices and some of them could satisfy the constraints that we have found. Anyway, even if this is not the case for these models, there are other 177 models that have not yet been analysed in detail.

#### 4. Phenomenology of this scenario

The main characteristic of the scenario studied in previous sections, is the presence at low energy of extra matter. In particular, we have obtained that three generations of Higgses and vector-like colour triplets are necessary.

Since more Higgs particles than in the MSSM are present, there will be of course a much richer phenomenology [45]–[48]. Note for instance that the presence of six Higgs doublets implies the existence of sixteen physical Higgs bosons, eleven of them are neutral and five charged. On the other hand, it is well known that dangerous flavour-changing neutral currents (FCNCs) may appear when fermions of a given charge receive their mass through couplings with several Higgs doublets [49]. This is because the transformations diagonalizing the fermion mass matrices do not, in principle, diagonalize the Yukawa interactions. This situation might be present here since we have three generations of supersymmetric Higgses. In general, the most stringent limit on flavour-changing processes comes from the small value of the  $K_L - K_S$  mass difference [50]. There are two approaches in order to solve this potential problem. In one of them one assumes that the extra Higgses are sufficiently massive making  $\Delta S = 2$  neutral currents small enough not to contradict the experimental data [51, 50, 52]. In this case the actual lower bound on Higgs masses depends on the particular texture chosen for the Yukawa matrices, but can be as low as 120–200 GeV [53]. In the other approach the Yukawa couplings have some symmetries eliminating FCNCs completely [49]. The simplest example is when the couplings between the extra Higgses and quarks of a given charge are forbidden. If the three Yukawa-coupling matrices are present, still one can avoid FCNCs if the matrices are proportional. In this case one can always choose a basis in which only one generation of Higgses couples to quarks. It would be very interesting to analyse which approach arises naturally in these orbifold models thanks to the stringy selection rules [54].

Concerning the three generations of vector-like colour triplets,  $D$  and  $\bar{D}$ , they should acquire masses above the experimental limit  $\mathcal{O}(200 \text{ GeV})$ . This is possible, in principle, through couplings with some of the extra singlets with  $Y = 0$ , say  $N_i$ , which are usually left at low energies, even after the Fayet-Iliopoulos breaking. For example, in the model of

ref. [16], there are 13 of these singlets. Thus couplings  $N_i D \bar{D}$  might be present. From the electroweak symmetry breaking, the fields  $N_i$  a VEV might develop. Note in this sense that the Giudice-Masiero mechanism to generate a  $\mu$  term through the Kähler potential is not available in prime orbifolds as  $Z_3$  [55]. Thus an interesting possibility to generate it, given the large number of singlets present in orbifold models, is to consider couplings of the type  $N_j H_u H_d$ . It is also worth noticing that some of these singlets might not have the necessary couplings to develop VEVs and then might be candidates for right-handed neutrinos.

Before concluding, a few comments about the hypercharges of the extra colour triplets are necessary. Of the three models that we have used as examples in the previous section, two of them, the ones with  $C^2 = 15/37$  and  $C^2 = 3/11$ , have triplets with non-standard fractional electric charge,  $\pm 1/15$  and  $\pm 1/6$  respectively. The existence of this kind of matter is a generic property of the massless spectrum of supersymmetric models [56, 57]. This means that they have necessarily colour-neutral fractionally charged states, since the triplets bind with the ordinary quarks. For example, the model with triplets with electric charge  $\pm 1/6$  will have mesons and baryons with charges  $\pm 1/2$  and  $\pm 3/2$ . On the other hand, the model with  $C^2 = 3/17$  has ‘standard’ extra triplets, i.e. with electric charges  $\mp 1/3$  and  $\pm 2/3$ ; these will therefore give rise to colour-neutral integrally charged states. For example, a  $d$ -like quark  $D$  forms states of the type  $u\bar{D}$ ,  $uuD$ , etc. These results are consistent with general arguments [58]: level-one string models can be modular-invariant and free of colour-neutral fractionally charged states if and only if

$$\frac{7}{12} + \frac{C^{-2}}{4} = 0 \pmod{1}. \quad (4.1)$$

It is trivial to see that only the model with  $C^2 = 3/17$  fulfils this condition.

Let us now briefly discuss if our assumption of light extra colour triplets to solve the unification problem is consistent with the above results. In principle, the existence of stable charged states creates conflicts with cosmological bounds. For example, thermal production of these particles would overclose the Universe unless their masses are below a few TeV [59, 60]. In models with ‘non-standard’ extra triplets, the lightest colour-neutral fractionally charged state, due to electric charge conservation, will be stable. However, since its mass must be of the order of the supersymmetric scale  $M_S$  to have unification, the above mentioned conflict is not present in our case. On the other hand, as pointed out in ref. [57], the estimation of its relic abundance contradicts limits on the existence of fractional charge in matter (less than  $10^{-20}$  per nucleon [60]). Thus, avoiding such fractionally charged states is necessary. A possible mechanism to carry it out is inflation. Inflation would dilute these particles, saving these unification models. The reheating temperature  $T_{RH}$  should be low enough not to produce them again. A recent calculation implies that  $T_{RH}$  must be smaller than  $10^{-3}$  times the mass of the particle [61], i.e.  $T_{RH} \lesssim 1$  GeV in our case. This is possible in principle, since the only constraint on this temperature is to be larger than 1 MeV not to spoil the successful nucleosynthesis predictions. Another possibility studied in the literature [62] to solve the problem is that the extra triplets transform also under a non-abelian group in the hidden sector. Then, they may be confined into integrally charged

states. However, in the orbifold models studied here this possibility is not available, since the extra triplets are always singlets under the non-abelian hidden groups.

Let us concentrate now on the ‘standard’ extra triplets. In addition to the possible mass terms,  $N\overline{D}\overline{D}$ , discussed above, these triplets could have FCNC couplings with ordinary quarks, e.g.  $H_d Q \overline{D}$ . Then decays of the  $D$ ’s through couplings with Higgses and quarks are possible, and therefore the colour-neutral integrally charged states will not be stable. Other decay channels are present in this case. It is well known that FCNC couplings may appear if all fermions with the same charge and helicity do not have the same  $SU(2)$  quantum numbers, because of their mixing through mass matrices [49, 63]. This is true even for one family of Higgses. As a consequence, the  $D$ ’s can also decay through charged and neutral currents. Thus models where this type of triplets are light ( $\sim M_S$ ) may unify the couplings at  $M_{\text{GUT}}$  without any cosmological problem, and will only be observed in collider experiments. Analyses of their production modes and decays will be similar to those of extra fermions in  $E_6$  theories [64]. Of course, there are further dangerous operators involving the  $D$ ’s, like  $QL\overline{D}$ ,  $\overline{u}d\overline{D}$ , etc., which must be forbidden by  $U(1)$  gauge invariance or stringy selection rules. Recall, as discussed below eq. (3.11), that light triplets with hypercharge  $\pm 2/3$  are not good to obtain the correct value for  $C^2$ , so only light  $D$ ’s with hypercharge  $\mp 1/3$  will be helpful. Note that the above model with  $C^2 = 3/11$  may also belong to this class since, in addition to triplets with electric charge  $\pm 1/6$ , it also has triplets with charge  $\mp 1/3$ , and these could be the light ones.

On the other hand, the colour-neutral integrally charged states are not automatically unstable in all models. The appropriate Yukawa couplings may be forbidden by  $U(1)$  gauge invariance or stringy selection rules, and therefore these states would be stable.

## 5. Final comments and outlook

We have attacked the problem of the unification of gauge couplings in Heterotic String constructions. Assuming that the Standard Model arises from the  $Z_3$  orbifold, we have obtained that  $\alpha_3$  and  $\alpha_2$  cross at the right scale, in a natural way, when a certain type of extra matter is present. In this sense three families of supersymmetric Higgses and vector-like colour triplets might be observed in forthcoming experiments. The unification with  $\alpha_1$  is obtained if the model has the appropriate normalization factor of the hypercharge. Our solution implies that the apparent unification of the MSSM using  $SU(5)$  normalization is just an accident, without any physical relevance.

Let us recall that although we have been working with explicit orbifold examples, our arguments are quite general and can be used for other schemes where the Standard Model gauge group with three generations of particles is obtained, since extra matter and anomalous  $U(1)$ ’s are generically present in compactifications of the Heterotic String. Even models with gauge groups larger than that of the Standard Model might be analyzed following the lines of this paper, after their breaking, using e.g. the Fayet-Iliopoulos term, to  $SU(3) \times SU(2) \times U(1)_Y \times G_{\text{hidden}}$ .

Once we have guessed the right phenomenological properties that, in our opinion, the candidates to the Superstring Standard Model must have, the next natural step consists

in searching explicitly the right model [65]. The main difficulty resides in how to obtain the observed structure of fermion masses and mixing angles. It is true that one can find interesting results in the literature. In particular, orbifold spaces have a beautiful mechanism to generate a mass hierarchy at the renormalizable level. Namely, Yukawa couplings can be explicitly computed and they get suppression factors, which depend on the distance between the fixed points to which the relevant fields are attached [37, 66]. These distances can be varied by giving different VEVs to the  $T$ -moduli associated to the size and shape of the orbifold. Of course, as usual in String Theory, it is extremely difficult to implement this mechanism in a particular model. However, we argue again that if the Standard Model arises from this type of constructions, there must exist one model where this can be done. Perhaps we may turn the above difficulty into a virtue, since the weird structure of Yukawas might be used as a hint to find the model.

If, at the end of the day, a model with the characteristics described above is found, this would be a great success. Of course it is not the end of the story, since in order to compute the explicit values of Yukawas we need to know the VEVs of the  $T$ -moduli. Unfortunately, these are related to the breaking of supersymmetry, and this is one of the biggest problems in String Theory. As a matter of fact we should also be able to compute the value of  $g_{\text{GUT}}$  obtained through phenomenological arguments in section 3. But, again, this is given by the VEV of another modulus field ( $S$ ), and therefore it is also connected with the mechanism of supersymmetry breaking. It is true that there are candidates for this task, such as gaugino condensation in a hidden sector, and that we have hidden gauge groups that could condensate. However, again, implementing this mechanism in a particular model is not easy.

Finally, the soft terms should be computed in order to connect the Supersymmetric Standard Model with the low-energy world. They should fulfil all kinds of phenomenological requirements. For instance, avoiding dangerous FCNCs is one of them.  $Z_3$  orbifold models with two Wilson lines automatically fulfil it, since the three generations of a given type of particle are associated to the same sector.

In conclusion, the task to be carried out in order to find the Superstring Standard Model is very hard, cumbersome, and in some sense, tedious, but indispensable if one wants to show that String Theory is the fundamental theory of particle physics.

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